

# Duolimpiadi

## Ancient problems

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### Lore

An almost unknown branch of archeology is so-called *mathemarcheology*, which studies mathematical problems posed by ancient populations. It's mathemarcheological convention, when a problem is translated from the original manuscript, to leave the variable names untouched. Here are collected four problems from various parts of the world, following the above convention. A little-known fact is that the ancient people used LaTeX. Mathemarcheologists, more civilized individuals, chose to use Typst instead; we can only follow in their footsteps.

## 1. The Parthian game

Vachagan (the Brave) e Shapur (son of Pabag) play a game: they write the number 2 on a blackboard, and then in turn, starting from Vachagan, erase the number  $\varsigma$  and write  $\varsigma + \omega$ , where  $\omega$  is a prime divisor of  $\varsigma$ . The first person to write a number on the board that's greater than 69 loses *the game*<sup>1</sup>. Assuming both players play perfectly, determine who wins the game.

## 2. The Emperor of Maurya's cube

Let  $\aleph, \square, \Lambda \in \mathbb{Z}^+$  for which  $\frac{\aleph}{\square} + \frac{\square}{\Lambda} + \frac{\Lambda}{\aleph} \in \mathbb{Z}$ . Prove that  $\aleph \square \Lambda$  is a perfect cube.

## 3. The Mycenaean grid

Determine all the  $\dagger \in \mathbb{Z}^+$  whose divisors can be inserted in a rectangular grid, each divisor in exactly one cell, where:

- the absolute difference between the sum of any two rows is at most 1
- the absolute difference between the sum of any two columns is at most 1

## 4. The phoenician function

Let  $\varkappa \in \mathbb{Z}^+$  be fixed. Determine as a function of  $\varkappa$  all the functions  $\gamma : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$  per cui, per ogni  $\gamma \in \mathbb{Z}^+$

- $\gamma(\gamma) < \gamma(\gamma + 1)$
- $\gamma^\varkappa(\gamma) = (\varkappa + 1)\gamma$ , where  $\gamma^\varkappa$  indicates  $\gamma$  applied  $\varkappa$  times.

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<sup>1</sup>So did you!